COMPUTATION OF STEADY-STATE TEMPERATURE FIELD IN TURBULENT GAS FLOW IN TWO-DIMENSIONAL CHANNEL WITH HEAT RELEASING WALLS

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The steady state temperature field in a turbulent channel flow is computed analytically taking into consideration the heat released from the dissipation of vortices and absorbed in the work by the pressure forces. The turbulent thermal conductivity is represented by a piecewise linear function of the transverse coordinate.

1. The temperature distribution in a two-dimensional channel with turbulent flow of the fluid has been investigated in detail for steady-state temperature profiles both for the temperature specified at the walls and for the heat releasing walls (for example, see [1]). However, apparently there have not been any investigations for determining the temperature taking into consideration the heat released from the internal friction in the gas and absorbed in the work by the pressure forces. The object of the present work was to determine the steady-state temperature field in a channel with heat releasing walls analytically taking this phenomenon into consideration. The thermophysical constants are assumed to be independent of temperature and the velocity is taken to be the average velocity over the transverse cross section of the channel. The turbulent thermal conductivity k_T is approximated by two straight lines [2, 3]:

$$K \equiv 1 + k_T k_M^{-1} = 1 + l_0 (1 - \xi) \quad \text{for} \quad \xi_0 \leqslant \xi \leqslant 1; \tag{1.1}$$

$$K = K_0 = 1 + l_0 (1 - \xi_0) \quad \text{for} \quad 0 \leqslant \xi \leqslant \xi_0. \tag{1.2}$$

In the immediate vicinity of the wall this distribution results in an enhanced total thermal conductivity, since there

$$k_T k_M^{-1} \sim (1 - \xi)^m, \quad m = 3 - 4.$$

However, this does not distort the temperature profile significantly, since at the wall itself molecular thermal conductivity predominates.

Using dimensionless quantities the heat conduction equation can be written in the following form taking into consideration the heat from the internal friction and the heat absorbed in the work by the pressure forces:

$$\bar{u} \frac{\partial \vartheta}{\partial \tau} = \frac{\partial}{\partial \xi} K \frac{\partial \vartheta}{\partial \xi} + \psi (\xi). \tag{1.3}$$

The boundary conditions at the entrance and at the walls, and the conditions of symmetry are the following:

$$\vartheta = 1 \quad \text{for} \quad \tau = 0, \tag{1.4}$$

$$\partial \vartheta / \partial \xi = -g \quad \text{for} \quad \xi = 1, \tag{1.5}$$

$$\partial \vartheta / \partial \xi = 0 \quad \text{for} \quad \xi = 0. \tag{1.6}$$

The matching conditions at $\xi = \xi_0$ are

$$\vartheta_1 = \vartheta_2, \quad \partial \vartheta_1 / \partial \xi = \partial \vartheta_2 / \xi. \tag{1.7}$$

Here and below the temperature for the segment $0 \leq \xi \leq \xi_0$ is denoted by ϑ_1 and the temperature for the segment $\xi_0 \leq \xi \leq 1$ by ϑ_2.

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We take Laplace transform of (1.3) taking (1.1), (1.2) into consideration:

$$\bar{u}(p\vartheta_1^* - 1) = K_0 d^2\vartheta_1^*/d\xi^2 + p^{-1}\psi(\xi), \tag{1.8}$$

$$(\xi - 1 - l_0^{-1})\, d^2\vartheta_2^*/d\xi^2 + d\vartheta_2^*/d\xi + \bar{u}pl_0^{-1}\vartheta_2^* = h(\xi), \tag{1.9}$$

where

$$h(\xi) = \bar{u}l_0^{-1} + \psi(\xi)(pl_0)^{-1}. \tag{1.10}$$

Equation (1.9) is a Bessel type equation and for h = 0 it has the following solution [4]

$$\vartheta_2^*(\xi) = Z_0(y(\xi)) \equiv AJ_0(y) + BY_0(y), \tag{1.11}$$

where

$$y(\xi) = 2\,[\bar{u}\alpha l_0^{-1}(1 + l_0^{-1} - \xi)]^{1/2}, \quad \alpha \equiv -p.$$

Knowing any homogeneous solution the particular solution can be found [4]. The complete solution is of the form

$$\vartheta_2^* = AJ_0(y) + BY_0(y) + \varphi_1 \int_{\xi_0}^{\xi} E^{-1}\varphi_1^{-2}\left(\int_{\xi_0}^{\xi} \varphi_1(\xi')\, h(\xi')\, d\xi'\right) d\xi, \tag{1.12}$$

where

$$E = \xi - 1 - l_0^{-1}, \quad \varphi_1 = J_0(y(\xi)). \tag{1.13}$$

We subject (1.12) to the condition at the wall (1.5):

$$\alpha^{-1}g = Z_1(y_1)\sqrt{\bar{u}\alpha} + J_1(y_1)\sqrt{\bar{u}\alpha}\int_{\xi_0}^{1} E^{-1}\varphi_1^{-2}\left(\int_{\xi_0}^{\xi}\varphi_1 h\, d\xi'\right) d\xi - l_0^{-1}\int_{\xi_0}^{1}\varphi_1 h\, d\xi / J_0(y_1). \tag{1.14}$$

Under the condition of symmetry (1.6) the solution of (1.8) is

$$\vartheta_1^* = C\varphi_2(\xi) + \varphi_2 E_0^{-1}\int_0^{\xi}\varphi_2^{-2}\left(\int_0^{\xi}\varphi_2 h\, d\xi'\right) d\xi, \tag{1.15}$$

where

$$E_0 = E(\xi_0), \quad \varphi_2 = \cos(\lambda_0 y_0 \xi/\xi_0).$$

The matching conditions (1.7) are formulated taking (1.12), (1.15) into consideration. Eliminating coefficient C from these we obtain together with (1.14) a system for determining the coefficients A and B; the solution of this system is

$$\begin{aligned} A &= \Delta^{-1}\{f_1[Y_1(y_0)\cos\lambda_0 y_0 + Y_0(y_0)\sin\lambda_0 y_0] - f_2 Y_1(y_1)\},\\ B &= \Delta^{-1}\{-f_1[J_1(y_0)\cos\lambda_0 y_0 + J_0(y_0)\sin\lambda_0 y_0] + f_2 J_1(y_1)\}. \end{aligned} \tag{1.16}$$

Here we have used the following notations:

$$\begin{aligned} \Delta &= J_1(y_1)[Y_1(y_0)\cos\lambda_0 y_0 + Y_0(y_0)\sin\lambda_0 y_0] - Y_1(y_1)[J_1(y_0)\cos\lambda_0 y_0 + J_0(y_0)\sin\lambda_0 y_0],\\ f_1 &= \alpha^{-1}g(\alpha\bar{u})^{-1/2} + l_0(\alpha\bar{u})^{-1/2}\int_{\xi_0}^{1}\varphi_1 h\, d\xi / J_0(y_1) - J_1(y_1)\int_{\xi_0}^{1} E^{-1}\varphi_1^{-2}\left(\int_{\xi_0}^{\xi}\varphi_1 h\, d\xi'\right) d\xi, \end{aligned} \tag{1.17}$$

$$f_2 = -2y_0^{-1}\int_0^{\xi_0}\varphi_2 h\, d\xi, \quad y_1 = y(1), \quad y_0 = y(\xi_0), \quad \lambda_0 = \frac{1}{2}\xi_0(1 + l_0^{-1} - \xi_0)^{-1}. \tag{1.18}$$

In order to obtain the temperature $\vartheta(\tau, \xi)$ we find the inverse transform of (1.12), (1.15). In particular, for determining the steady-state temperature field it is necessary to have the expansion of the obtained solution in the neighborhood of $\alpha = 0$. Since the function $\vartheta_2^*(\alpha)$ has a pole of multiplicity 1 and 2 at $\alpha = 0$, in the expansion of $\Delta(\alpha)$ it is sufficient to consider terms $0(1)$ and $0(\alpha)$. The expansion of Δ gives

$$\pi\Delta = A_0 + B_0\alpha + 0(\alpha^2), \tag{1.19}$$

where

$$A_0 = l_0 b_0^{-1}, \quad B_0 l_0^2/\bar{u} = 2b_0[(1 + 2\lambda_0)\ln b_0 + \lambda_0 + \lambda_0^2] + 1/2b_0$$

$$-2b_0^{3/2}\left(\frac{1}{4}+\frac{2}{3}\lambda_0^3+\lambda_0^2+\lambda_0\right),\quad b_0=(1+l_0-l_0\xi_0)^{1/2}. \tag{1.20}$$

The use of the two necessary conditions [1) for zero sources ($g = \psi = 0$) we must have $\vartheta_2^* = -\alpha^{-1}$; 2) terms containing $h_0 = -\psi/\alpha l_0$ have only first order poles at $\alpha = 0$, since $\int_0^1 \psi d\xi = 0$] gives the desired expansion of the solution $\vartheta_2^*(\alpha, \xi)_{\alpha\to 0}$. Taking the inverse transform in this expansion we obtain the asymptotic temperature distribution ϑ_2^0 for $\tau \to \infty$ after computing the residues at the point $p = 0$ (practically for $\tau \geq 0.005$):

$$\frac{1-\vartheta_2^0}{g}=\frac{\tau}{\bar{u}}-\frac{1}{l_0}\left(1+\frac{1}{l_0}\right)\left(1-g^{-1}\int_{\xi_0}^{1}\psi d\xi\right)\ln\frac{1+l_0^{-1}-\xi}{1+l_0^{-1}-\xi_0}+B_1$$
$$-\frac{\xi-\xi_0}{l_0}-g^{-1}l_0^{-2}\int_{\xi_0}^{1}E^{-1}\left(\int_{\xi_0}^{\xi}\psi d\xi'\right)d\xi-g^{-1}l_0^{-1}\int_{\xi_0}^{\xi}E^{-1}\left(\int_{\xi_0}^{\xi}\psi d\xi'\right)d\xi \tag{1.21}$$
$$-(1+l_0^{-1})\xi_0 b_0^{-2}g^{-1}\int_{\xi_0}^{1}\psi d\xi+l_0^{-2}g^{-1}\ln(1+l_0-\xi l_0)\int_0^{\xi_0}\psi d\xi+g^{-1}\left(l_0^{-1}\int_{\xi_0}^{1}\xi\psi d\xi-\int_0^{\xi_0}\xi^2\psi d\xi/2b_0^2\right),$$

where

$$B_1=\frac{1}{2}l_0^{-3}+2l_0^{-2}(1+l_0^{-1})\left[\ln b_0+\lambda_0(1+\lambda_0)(1+2\lambda_0)^{-1}\right]$$
$$-(1+l_0^{-1/2})^2 l_0^{-1}(1+2\lambda_0)^{-2}\left[\frac{1}{2}+2\lambda_0(1+\lambda_0)+\frac{4}{3}\lambda_0^3\right]+\lambda_0\xi_0 l_0^{-1}.$$

The solution depends on the parameters ξ_0, l_0, and the function ψg^{-1}.

The coefficient C is determined from the matching conditions and the temperature in the transformed plane ϑ^* becomes

$$\vartheta_1^*=Z_0(y_0)\varphi_2(\xi)/\varphi_2(\xi_0)-\varphi_2(\xi)E_0^{-1}\int_{\xi}^{\xi_0}\varphi_2^{-2}\left(\int_0^{\xi}\varphi_2 h d\xi'\right)d\xi. \tag{1.22}$$

Using the expansion of $\vartheta_2^*{}_{\alpha\to 0}$ we find the required expansion of function (1.22):

$$\vartheta_1^*{}_{\alpha\to 0}=\vartheta_2^*{}_{\alpha\to 0}(\xi_0)-\frac{g}{2\alpha b_0^2}(\xi_0^2-\xi^2)+(E_0 l_0\alpha)^{-1}\int_{\xi}^{\xi_0}\left(\int_0^{\xi}\psi d\xi'\right)d\xi. \tag{1.23}$$

Hence we obtain

$$\frac{1-\vartheta_1^0}{g}=\frac{1-\vartheta_2^0(\tau,\ \xi_0)}{g}-\frac{\xi_0^2-\xi^2}{2b_0^2}-(gb_0^2)^{-1}\int_{\xi}^{\xi_0}\left(\int_0^{\xi}\psi d\xi'\right)d\xi. \tag{1.24}$$

2. In solution (1.21), (1.22) let us introduce the explicit dependence of function ξ on the coordinate

$$\psi(\xi)=-\frac{h^2}{k_M(T_в-T_0)}\left(u\frac{\partial P}{\partial x}+\rho\nu_т\left(\frac{\partial u}{\partial y}\right)^2\right). \tag{2.1}$$

Assuming that the rate of production of the turbulent energy is equal to the rate of its dissipation into heat and also that the velocity u has a power law dependence $u = u_m(1-\xi)^{1/n}$, we determine the turbulent viscosity from the formula [2] $\nu_T = -hu_*^2\xi/u_m d\tilde{u}/d\xi$; then making use of the formula $-dP/dx = \tau_0 h^{-1} \equiv \rho u_*^2/h$, where the dynamic velocity $u_* = u_m\sqrt{\lambda/8}/(1 + 3.75\sqrt{\lambda/8})$, we obtain

$$\psi=\tilde{\varepsilon}_т(1-\xi)^{1/n}\left(1-\frac{\xi n^{-1}}{1-\xi}\right). \tag{2.2}$$

Here

$$\tilde{\varepsilon}_т=(\gamma-1)\theta_0^{-1}M_0^2\mathrm{Re}\,\mathrm{Pr}\,(\lambda/8)/(1+3.75\sqrt{\lambda/8})^2. \tag{2.3}$$

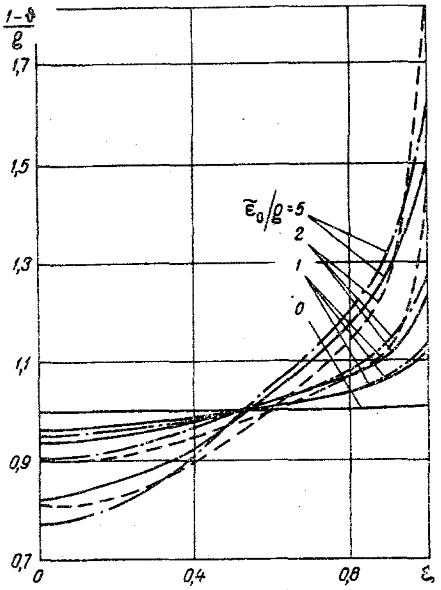

Fig. 1. Distribution of dimensionless temperature in the channel for $\tau/\bar{u} = 1$, $l_0 = 400$, $n = 7$, $\xi_0 = 0.75$ for different values of the parameters $\tilde{\varepsilon}_0/g$ and δ. Dashes, $\delta = 0$; continuous curve, 0.005; dash-dots, 0.0025.

For Blasius' law of friction

$$\tilde{\varepsilon}_T = 0.023\,(\gamma - 1)\,\theta_0^{-1}\,\mathrm{Pr}\,\mathrm{Re}^{3/4}. \tag{2.4}$$

The expression for ψ (2.2) satisfies the condition

$$\int_0^1 \psi d\xi = 0. \tag{2.5}$$

A comparison with Laufer's experiments (see [2]) showed that the production of turbulent energy, given by the second term in (2.2), corresponds to the experimental data everywhere except in the immediate vicinity of the wall ($y_+ < 5$; $\xi > 1-\delta$).

We express constant l_0 in terms of the flow parameters. Let us put $\varphi_0 = \nu_T/u_* h$, $\mathrm{Pr}_T = c_p \eta_T k_T^{-1}$, $\mathrm{Pr} = c_p \eta_M k_M^{-1}$. Hence $k_T/k_M = K-1 = \varphi_0 u_* h \nu^{-1} \mathrm{Pr}\,\mathrm{Pr}_T^{-1}$. Expressing the dynamic velocity in terms of the friction coefficient considering the adopted approximation $\varphi_0 = \varepsilon_{00}(1-\xi)$, where $\varepsilon_{00} = 0.07(1-\xi_0)^{-1}$, and comparing with (1.1) we obtain

$$l_0 = \varepsilon_{00}\,\mathrm{Re}\,\mathrm{Pr}\,\mathrm{Pr}_T^{-1}\sqrt{\lambda/8}\,(1 + 3.75\sqrt{\lambda/8})^{-1}. \tag{2.6}$$

From the comparison of $\tilde{\varepsilon}_T$ and l_0 we obtain $\tilde{\varepsilon}_T = \tilde{\varepsilon}_0 k_0 l_0$, where $\tilde{\varepsilon}_0 = \theta_0^{-1} M_0^2(\gamma-1)\mathrm{Pr}_T$, $k_0 = \sqrt{\lambda/8}\,\varepsilon_{00}^{-1}(1 + 3.75\sqrt{\lambda/8})^{-1}$.

Substituting function ψ (2.2) into solution (1.21), (1.24) we get

$$\frac{1-\vartheta_1^0}{g} = \frac{1-\vartheta_2^0(\tau,\,\xi_0)}{g} - \frac{\xi_0^2-\xi^2}{2(l_0+1-\xi_0 l_0)} - \frac{\tilde{\varepsilon}_0 k_0 n}{g(1+l_0^{-1}-\xi_0)}\left[(1-\xi)^{1+n^{-1}}\left(\frac{1}{n+1} - \frac{1-\xi}{2n+1}\right) - (1-\xi_0)^{1+n^{-1}}\left(\frac{1}{n+1} - \frac{1-\xi_0}{2n+1}\right)\right], \tag{2.7}$$

$$\begin{gathered}\frac{1-\vartheta_2^0}{g} = \frac{\tau}{\bar{u}} - \frac{1}{l_0}\left(1+\frac{1}{l_0}\right)\ln\frac{1+l_0^{-1}-\xi}{1+l_0^{-1}-\xi_0} + B_1 \\ - \frac{\xi-\xi_0}{l_0} + \frac{\tilde{\varepsilon}_0 k_0}{g}\Big[(2\lambda_0^2-1)\,\xi_0(1-\xi_0)^{1+n^{-1}} + I_1(\xi) - l_0^{-1} I_1(1) \\ + 2l_0^{-1}\lambda_0^2\xi_0(1-\xi_0)^{1/n} + \left(\frac{1}{n+1} - \frac{n+1}{2n+1}(1-\xi_0)\right)(1-\xi_0)^{1+n^{-1}} - \frac{1}{2}(1+l_0^{-1}-\xi_0)^{-1} I_2\Big].\end{gathered} \tag{2.8}$$

Here

$$\begin{gathered}I_1(\xi) = \int_{\xi_0}^{\xi} \xi(1+l_0^{-1}-\xi)^{-1}(1-\xi)^{1/n}d\xi = n(1-\xi_0)^{1/n} \\ \times\Big[{}_2F_1(1,\,-n^{-1},\,1-n^{-1},\,z_1^{-1}) - \frac{1-\xi_0}{n+1}\,{}_2F_1(1,\,-1-n^{-1},\,-n^{-1}, \\ z_1^{-1})\Big] - n(1-\xi)^{1/n}\Big[{}_2F_1(1,\,-n^{-1},\,1-n^{-1},\,z^{-1}) - \frac{1-\xi}{n+1}\,{}_2F_1(1,\,-1-n^{-1},\,-n^{-1},\,z^{-1})\Big],\end{gathered} \tag{2.9}$$

where $z = -l_0(1-\xi)$, $z_1 = -l_0(1-\xi_0)$, ${}_2F_1$ is hypergeometric function. Expanding (2.9) in powers of z_1^{-1}, z^{-1}, for $|z| > 5$ we obtain

$$\begin{gathered}I_1 = n(1+l_0^{-1})\left[(1-\xi_0)^{1/n} - (1-\xi)^{1/n}\right] - \frac{n}{1+n}\left[(1-\xi_0)^{1+n^{-1}}\right. \\ \left. - (1-\xi)^{1+n^{-1}}\right] + \frac{n}{l_0(n-1)}\left[(1-\xi_0)^{-1+n^{-1}} - (1-\xi)^{-1+n^{-1}}\right] \\ + nl_0^{-2}(1-\xi)^{-2+n^{-1}}\left(\frac{1}{2n-1} - \frac{1-\xi}{n-1}\right) + 0(z_1^{-2},\,z^{-3}).\end{gathered} \tag{2.10}$$

For $|z| \le 5$ we can use the representation of the second term in (2.9) in terms of z.

Next

$$I_2 = \tilde{\varepsilon}_T^{-1}\int_0^{\xi_0}\xi^2\psi d\xi = \frac{2}{n+1} - \frac{2n+3}{2n+1} + \frac{n+1}{3n+1} + (1-\xi_0)^{1/n}$$
$$-(1-\xi_0)^{1+n^{-1}}\left[\frac{n+3}{n+1} - \frac{2n+3}{2n+1}(1-\xi_0) + \frac{n+1}{3n+1}(1-\xi_0)^2\right]. \quad (2.11)$$

3. The function ψ (2.2) used in Sec. 2 leads to unreal large values of $|\psi|$ at the wall giving overestimated value of the temperature. In order that ψ correspond to the experimental data in the entire range $0 \le \xi \le 1$ and satisfy condition (2.5) we take

$$\psi = \tilde{\varepsilon}_T(1-\xi)^{1/n}\left(A - \frac{\xi n^{-1}}{1-\xi}\right) \quad \text{for} \quad 0 \leqslant \xi < 1-\delta; \quad (3.1)$$

$$\psi = 0 \quad \text{for} \quad 1-\delta \leqslant \xi \leqslant 1, \quad (3.2)$$

where

$$A = (1+n^{-1})(1-\delta^{1/n})(1-\delta^{1+n^{-1}})^{-1} - n^{-1}; \; A \leqslant 1.$$

The dimensionless coordinate δ corresponds to the universal coordinate $y^+ = 5$: $\delta = 5\mathrm{Pr}\varepsilon_{00}/l_0\mathrm{Pr}_T$.

We make use of (3.1), (3.2) for evaluating the integrals in (1.21):

$$\int_{\xi_0}^{1}\psi d\xi = \tilde{\varepsilon}_T[\delta^{1/n}(1-\beta_1\delta) - (1-\xi_0)^{1/n}(1-\beta_1(1-\xi_0))],$$

$$I_3(1) = -\int_{\xi_0}^{1}E^{-1}\left(\int_{\xi_0}^{\xi}\psi d\xi'\right)d\xi = I_3(1-\delta) + \tilde{\varepsilon}_T[\delta^{1/n}(1+\beta_1\delta) - (1-\xi_0)^{1/n}(1+\beta_1\xi_0 - \beta_1)]\ln(1+\delta l_0),$$

$$\int_0^{\xi_0}(\xi-1)\psi d\xi = \tilde{\varepsilon}_T(1-\xi_0)^{1+n^{-1}}\left\{\frac{1}{1+n}\left[1-\left(\frac{\delta}{1-\xi_0}\right)^{1+n^{-1}}\right]\right.$$
$$\left. - \frac{An+1}{2n+1}(1-\xi_0)\left[1-\left(\frac{\delta}{1-\xi_0}\right)^{2+n^{-1}}\right]\right\},$$

$$\int_0^{\xi_0}\xi^2\psi d\xi = \tilde{\varepsilon}_T\left\{\frac{An+1}{3n+1} - \frac{2An+3}{2n+1} + \frac{An+3}{n+1} - 1 + (1-\xi_0)^{1/n}\right.$$
$$\left. -(1-\xi_0)^{1+n^{-1}}\left[\frac{An+3}{n+3} - \frac{(1-\xi_0)(2An+3)}{2n+1} + \frac{(1-\xi_0)^2(An+1)}{3n+1}\right]\right\},$$

$$\underset{(\xi<1-\delta)}{I_3(\xi)} = \tilde{\varepsilon}_T(1-\xi_0)^{1/n}(1-\beta_1+\xi_0\beta_1)\ln\frac{1+l_0^{-1}-\xi}{1+l_0^{-1}-\xi_0} + \tilde{\varepsilon}_T I_4(\xi),$$

$$\underset{(|z|>5)}{I_4(\xi)} = n[(1-\xi_0)^{1/n} - (1-\xi)^{1/n}] - \frac{n\beta_1}{n+1}[(1-\xi_0)^{1+n^{-1}} - (1-\xi)^{1+n^{-1}}]$$
$$-n(1-\xi_0)^{1/n}z_1^{-1}[(n-1)^{-1} + \beta_1(1-\xi_0)] + n(1-\xi)^{1/n}z^{-1}[(n-1)^{-1}$$
$$+\beta_1(1-\xi)] + z^{-2}n(1-\xi)^{1/n}\left(\frac{1}{2n-1} - \beta_1\frac{1-\xi}{n-1}\right) + 0(z_1^{-2}, z^{-3}).$$

For $0 \le \xi \le \xi_0$

$$\vartheta_1^0 = \vartheta_2^0(\tau, \xi_0) + \frac{g}{2}\frac{\xi_0^2-\xi^2}{b_0^2} - \tilde{\varepsilon}_0 k_0(1+l_0^{-1}-\xi_0)^{-1}\left\{(1-\beta_1)(\xi_0-\xi)\right.$$
$$\left. + \frac{n}{1+n}[(1-\xi_0)^{1+n^{-1}} - (1-\xi)^{1+n^{-1}}] - \frac{n\beta_1}{2n+1}[(1-\xi_0)^{2+n^{-1}} - (1-\xi)^{2+n^{-1}}]\right\}.$$

4. From an analysis of the expression (2.8) for the temperature we note that the temperature is determined mainly by the term $\tilde{\varepsilon}_0 k_0 I_1 g^{-1}$ with the factor l_0^{-1} removed; $I_1(\xi)$ increases rapidly as the wall is approached (for $\xi \to 1$). The index n is expressed [2] in terms of the coefficient of friction $n = \lambda^{-1/2}$, from which we get $k_0 \simeq n^{-1}$ for $\xi_0 = 0.75$.

The results of computations carried out with formulas (2.7), (2.8) ($\delta = 0$) for $l_0 = 400$ (Re $= 4.5 \cdot 10^4$, $\mathrm{Pr}/\mathrm{Pr}_T = 0.75$), $\xi_0 = 0.75$, $\theta_0^{-1} = 4$, $n = 7$, $\tilde{\varepsilon}_0 g^{-1} = 0$, 1/2, 1, 2, 5, are shown in Fig. 1. The temperature increases appreciably at the wall and falls off in the central part of the channel compared to the case $\tilde{\varepsilon}_0 g^{-1} = 0$. The assumed thermal conductivity (1.1) results in a logarithmic dependence of the temperature near the wall for $\psi = 0$. The temperature computed from the formulas of Sec. 3 with the same values of the parameters and $\delta = 0.005,\ 0.0025$ are also shown in the same figure. The values of the temperature near the wall are lower here.

5. Let us determine the Nusselt number

$$\mathrm{Nu} = h\,(\partial T/\partial y)_h\,(T_h - T_{м})^{-1} = -g\left(\vartheta^0_{\xi=1} - \int_0^1 \vartheta^0 d\xi\right)^{-1}.$$

Substituting (1.21), (1.24) we obtain

$$\mathrm{Nu} = 2l_0'\,[(1 + 2l_0^{-1} - l_0^{-2})\ln b_0 - (1-\xi_0)(5 - \xi_0 + 2l_0^{-1}) + 2\xi_0^3 l_0/3b_0^2]^{-1}. \tag{5.1}$$

Computations from formula (5) for $\mathrm{Pr} = 0.8$, $\mathrm{Pr_T} = 1$, $\mathrm{Re} = 10^4$–10^5 gives values of Nu close to the experimental values found from the formula $\mathrm{Nu} = 0.021 \cdot \mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}$ (higher than these by 5-10%). However, (5.1) does not give the experimental dependence $\mathrm{Nu} \sim \mathrm{Pr}^{1/3}$. In order to obtain this dependence it is necessary to assume a nonlinear profile $K(\xi)$ near the wall.

NOTATION

b	is the thickness of the thin wall of the channel thermally insulated from outside;
c_p	is the specific heat;
G	is the power density of the heat sources in the walls;
$2h$	is the width of the channel;
J_i, Y_i	are the Bessel functions of the first and second kind;
k_M, k_T	are the molecular and turbulent thermal conductivities;
p	is the variable in the transformed plane;
P	is the pressure;
T	is the temperature;
T_0, T_n	are the temperatures at the entrance to the channel and in the nominal state ($T_n > T_0$);
u	is the longitudinal flow velocity;
$\tilde{u} = u/u_m$;	
u_m	is the maximum velocity;
u_*	is the dynamic velocity;
x, y	are the coordinates along and transverse to the channel;
ox	is reckoned from the front section of the heated segment;
oy	is reckoned from the midpoint of the channel;
$\alpha = -p$;	
$\gamma = c_p/c_v$;	
$\eta_T = \rho\nu_T$	is the coefficient of turbulent viscosity;
ν	is the kinematic viscosity;
λ	is the coefficient of friction (number Re occurring in λ is determined by the hydraulic diameter $D = 4h$);
M_0	is the Mach number;
ρ	is the density;

$g = Ghb/k_M(T_n - T_0)$;
$\vartheta = (T_n - T)/(T_n - T_0)$;
$\theta_0 = (T_n - T_0)T_0^{-1}$, $\xi = y/h$;
$\tau = x/h\,\mathrm{Pr}\,\mathrm{Re}$;
$\mathrm{Re} = u_m/h\nu$;
$\bar{u} = (hu_m)^{-1}\int_0^h u\,dy$;
$y_+ = u_*(h-y)/\nu$;
$\beta_1 = (An + 1)(n + 1)^{-1}$.

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